ground water

Drawdown Distribution in the Vicinity of Nonvertical Wells

by Dennis E. Williams

Abstract

Recent developments in subsurface intake systems for ocean desalination plants are considering use of angled wells (slant wells) completed in permeable materials beneath the ocean floor. Conventional drawdown equations for vertical or horizontal wells are inadequate to properly describe the drawdown distribution in the vicinity of slant wells. Using the principle of superposition combined with standard well hydraulics, universal drawdown equations (UDE) are presented which calculate the drawdown distribution in the vicinity of production wells with inclination angles ranging from 0° (horizontal wells) to 90° (vertical wells). The method is computationally simple and other than the normal assumptions for standard well equations, it only requires that the calculated drawdown represent the drawdown which would be measured in a fully penetrating observation well. Solutions using the UDE are developed for confined, unconfined and semi-confined (leaky) aquifers and compared with analytical equations for vertical and horizontal wells, and with a numerical model for slant wells. The UDE is also applied to pumping test data from the Dana Point slant well project in Southern California.

Introduction

Drawdown equations in the vicinity of nonvertical wells were developed out of the necessity to calculate the water level distribution in the vicinity of angled wells (i.e., slant wells) extending beneath the ocean floor. Slant wells are increasingly being considered as viable methods to provide feed water supplies to ocean desalination plants (GEOSCIENCE 2004; Williams 2011). Previous investigators Theis (1935), Jacob (1940), Hantush (1964), Jacob (1946), Hantush and Papadopulos (1962), Kawecki (2000), and Kawecki et al. (2005) developed analytical solutions for drawdown around vertical and horizontal wells. However, these conventional vertical and horizontal well drawdown equations are inadequate to properly describe drawdown in the vicinity of slant wells. Zhan et al. (2002) and Hunt (2006) presented methods which included calculation of angled well drawdown. However, their methods are difficult to use in practice and require cumbersome type curves or access to computer programs. The drawdown equations presented herein are universal in that they may be used to calculate drawdown in the vicinity of production wells at any inclination angle below horizontal ranging from vertical to horizontal. These universal drawdown equations (UDE) merely superimpose drawdown solutions for a discrete number of point sinks placed within the vertical projection of the well screen interval. Drawdown solutions using the UDE are developed for confined aquifers, unconfined aquifers and semi-confined (leaky) aquifers for both steady and nonsteady flow conditions. However, in reality, none of the equations are truly universal but all are presented for completeness. For most problems of practical interest the nonsteady state confined aquifer equation (Jacob equation) would best fit the definition of the UDE.

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Received May 2012, accepted October 2012

doi: 10.1111/gwat.12000

Fundamental Equations

The drawdown distribution in the vicinity of an angled well is presented as the algebraic sum of drawdowns for a finite number of point sinks distributed along the vertical projection of the well screen. Each of the sinks has a fractional discharge rate proportional to the total well discharge rate and number of sinks. The UDE calculates the drawdown which would occur in a fully penetrating observation well removing any effects of partial penetration no matter how close to the pumping well (Hantush 1961). On one extreme, the inclination angle below horizontal is 0° and the horizontal projection of the well screen is the full length of the screen. In this case, the standard collector well equations for one lateral apply as described by Hantush-Papadopulos (1962). On the other extreme, the inclination angle is 90° below horizontal and the well is vertical with the horizontal projection of the well screen interval being a single point on the surface. In this case, the standard vertical well equations apply (Theis 1935; Jacob 1946). The number of sinks required for a smooth drawdown curve is primarily a function of the production well inclination angle, well screen length and radial distance from the well to the point where drawdown is calculated. However, other factors such as well and aquifer parameters influence this to a certain extent. For most problems of practical interest, a relatively few number of sinks are required for an adequate drawdown solution. A measure of the UDE accuracy is made using the "discretization error" relating the number of sinks and the radial distance to the point where drawdown is desired.

Basic Notation and Coordinate System

Figure 1 is a generalized cross section showing the three basic well types, vertical, horizontal, and angled (slant) as well as notations used in the UDE. Wells typically consist of a blank casing section and a perforated section (well screen) within the production zone of the aquifer.

Figure 2 is a plan view of an angled extraction well showing coordinate system notation. Coordinates for the well head (XW, YW) are shown relative to the origin of wellfield coordinates (X0, Y0). A number of point sinks are placed within the vertical projection of the well screen (XS) with the interval between the sinks denoted by (δ). The distances to each of the sinks from a desired drawdown point (X2, Y2) is also shown. The distances from each of the ith sinks to the drawdown point in question is shown as RP_i. Quadrant convention begins with quadrant I (0°–90°) and successive quadrants increasing in a clockwise direction.

UDE for Nonsteady-State Flow in a Confined Aquifer

Although the UDE can be developed to incorporate any of the standard well equations (i.e., nonsteady and steady state, unconfined, semi-confined or confined aquifers), the most applicable UDE is derived using a modification of the Jacob equation. Cooper and Jacob (1946) developed a simplified approximation to the drawdown distribution around nonleaky confined aquifers. Known as the "Jacob modified non-equilibrium method" an approximation to the Theis nonequilibrium method was made by truncation of an infinite series expression for small values of the variable u ($r^2S/4Tt$). The simplicity of the Jacob equation lies in the ease of calculation and does not rely on complex tables, type curve relationships or computer programs. The UDE incorporating the Jacob approximation results in a nonvertical well drawdown that is computationally simple and applicable to most problems of practical interest. Derivations for other flow regimes and aquifer types are summarized in Table 1 with more detail presented in Appendix S1. Other than the normal assumptions which apply to the Theis nonequilibrium equation and the Jacob equation, the assumptions required for UDE development include simulating production from the well by a finite number of point sinks. The sinks are spaced equally within the vertical projection of the well screen with each sink having a fractional discharge rate proportional to the total production of the well and number of sinks. The drawdown calculated by the UDE at any point in the flow field represents the drawdown which would be measured in a fully penetrating observation well.

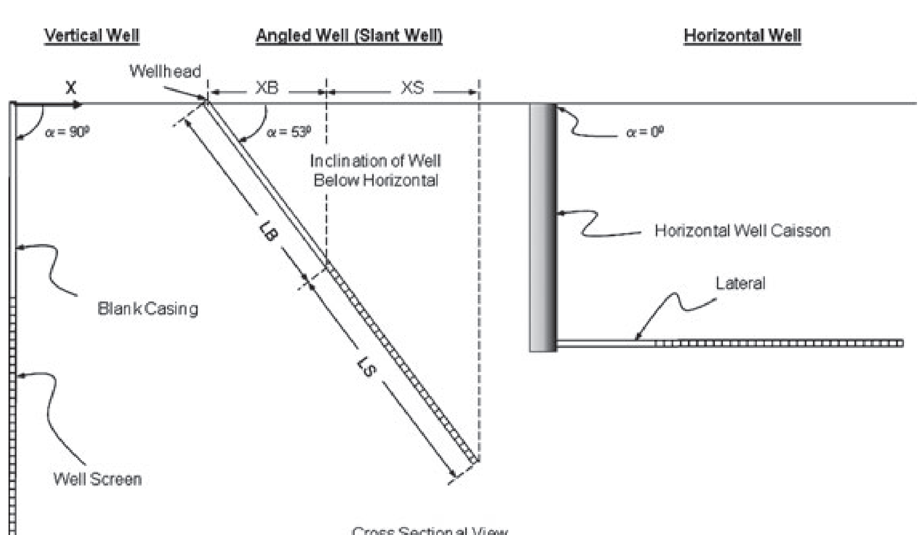

Figure 1. Basic well types.

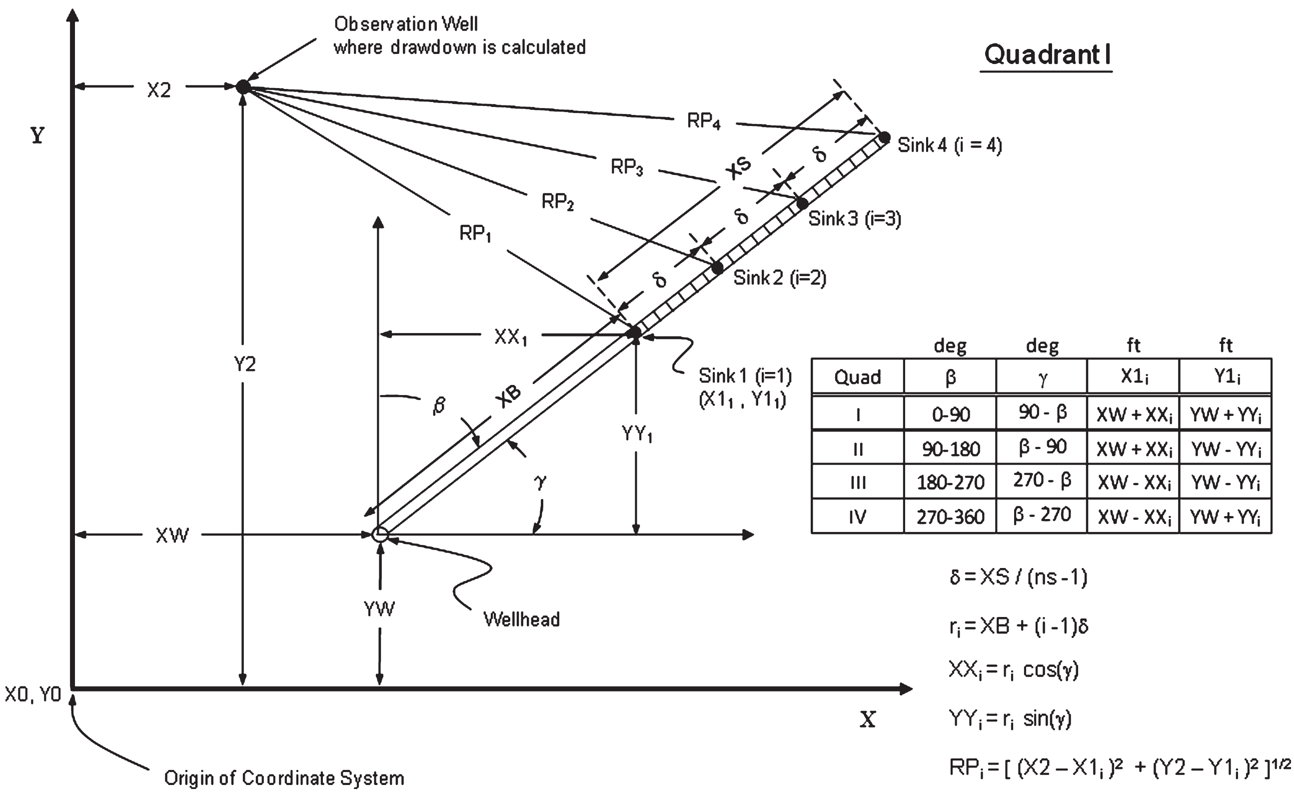

Figure 2. Plan view of angled well.

As such, any partial penetration effects from the sinks are eliminated (Hantush 1961). In addition, the assumption of a uniform inflow distribution (Hantush and Papadopulos 1962) is not necessary in the UDE solution and accurate drawdowns may be obtained even at close distances to the pumping well.

Jacob's equation for the drawdown around a vertical well for nonsteady-state flow in confined aquifers can be written as:

$$s = (264Q/T)\ \log[(0.3Tt)/(r^2S)] \qquad (1)$$

where s is vertical well drawdown, ft (m); Q, well discharge rate, gpm (L/s); T, aquifer transmissvity, gpd/ft (m^2/d); t, time since pumping started, d; r,radial distance from the pumping well, ft (m); and S, aquifer storativity, fraction.

In SI units

$$s = (15.83Q/T)\ \log[(2.25Tt)/(r^2S)]$$

An approximation to Equation 1 for nonvertical wells can be made by simulating the vertical projection of the production well screen by a number of point sinks. The drawdown can then be represented by:

$$s = \sum_{i=1}^{i=\text{ns}} s_i \qquad (2)$$

where s is total drawdown in the angled well, ft, (m) and s_i, incremental drawdown from the ith sink, ft, (m).

Rearranging Equation 1 for the drawdown from an individual sink:

$$\text{si} = (264\ \text{qi/T})\ [\log(0.3\text{Tt/S}) - 2\log(\text{RPi})] \qquad (3)$$

where q_i is discharge rate of ith sink $= Q/\text{ns}$, gpm (L/s); ns, number of point sinks along the vertical projection of the well screen; and RP$_i$, distance from observation well to ith sink, ft (m).

Letting$A = 264Q/(\text{ns}\ T)$ and $B = (0.3Tt/S)$ then

$$s_i = A[\log(B) - 2\log(\text{RP}_i)] \qquad (4)$$

Combining Equations 2 and 4 results in

$$s = \text{ns}A\ log(B) - 2A\ \log(\text{RP}_1 \times \text{RP}_2 \times \text{RP}_3 \times \cdots \times \text{RP}_{\text{ns}}) \qquad (5)$$

Rearranging Equation 5 and combining parameters in the constants A and B results in

$$s = (264Q/T)\ [log(0.3Tt/S) - (2/\text{ns})\ \log(\text{RP}_1 \times \text{RP}_2 \times \text{RP}_3 \times \cdots \times \text{RP}_{\text{ns}})] \qquad (6)$$

In SI units (L/s, m, m^2/d)

$s = (15.83Q/T)\ [\log(2.25Tt/S) - (2/\text{ns})\ \log\ (\text{RP}_1 \times \text{RP}_2 \times \text{RP}_3 \times \cdots \times \text{RP}_{\text{ns}})]$

Equation 6 is the nonsteady state UDE which applies to nonvertical wells completed in confined aquifers.

Table 1
UDE Drawdown Equations for Various Aquifer Types and Flow Regimes

UDE Equation	Standard Well Equation	Aquifer Type	Flow Regime
$s = (264Q/T)\ [\log(0.3Tt/S) - (2/\text{ns})\log(\text{RP}_1 \times \text{RP}_2 \times \text{RP}_3 \times \cdots \times \text{RP}_{\text{ns}})]$	Jacob	Confined	Nonsteady
$s = [(114.6\ Q)/(T\text{ns})]\ [W(u_1) + W(u_2) + W(u_3) + \cdots + W(u_{\text{ns}})]$	Theis	Confined	Nonsteady
$s = [(114.6\ Q)/(T\text{ns})][W(u_1, r_1/B) + W(u_2, r_2/B) + W(u_3, r_3/B) + \cdots + W(u_{\text{ns}}, r_{\text{ns}}/B)]$	Hantush-Jacob	Semi-Confined	Nonsteady
$s = [(114.6\ Q)/(T_0\ \text{ns})]\ [(1+\text{CF}_1)\ W(\rho_1^2/4\tau) + (1+\text{CF}_2) \times W(\rho_2^2/4\tau) + (1+\text{CF}_3)\ W(\rho_3^2/4\tau) + \cdots + (1+\text{CF}_{\text{ns}}) W(\rho_{\text{ns}}^2/4\tau)]$	Boulton	Unconfined	Nonsteady
$s = (528\ Q/T)\ [\log(r_0) - (1/\text{ns})\log(r_1 \times r_2 \times r_3 \times \cdots \times r_{\text{ns}})]$	Thiem	Confined	Steady
$s = [(229\ Q)/(T\ \text{ns})]\ [K_0(r_1/B) + K_0(r_2/B) + K_0(r_3/B) + \cdots K_0(r_{\text{ns}}/B)]$	Hantush	Semi-Confined	Steady
$s_i' = [(528\ Q)/(K\ D_0\ \text{ns})]\ \log(r_0/r_i)$	Dupuit-Forcheimer	Unconfined	Steady
$s' = s - (s^2/2\ D_0)$—Jacob's correction			
(Note: the Thiem equation may be used if Jacob's correction is applied)			

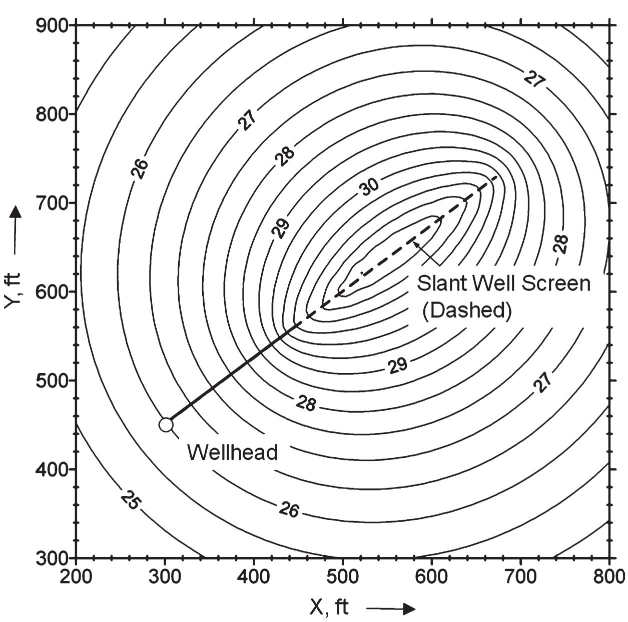

Figure 3. Drawdown distribution (ft) in the vicinity of a slant well calculated using Equation 6. $\alpha = 23°$, $\beta = 53°$, $L = 500$ ft, LS = 300 ft, $K = 500$ gpd/ft^2, $S_s = 1 \times 10^{-6}$/ft, $b = 117$ ft, $Q = 1000$ gpm, $t = 365$ d, ns = 10, XW = 300 ft, YW = 450 ft.

UDE for Other Aquifer Types and Flow Regimes

Table 1 summarizes UDE drawdown solutions for various aquifer types and flow conditions. Details regarding development of each solution may be found in Appendix S1.

Example Calculation

Figure 3 shows an example calculation of the drawdown distribution in the vicinity of a slant well in quadrant I using the nonsteady-state UDE equation for confined aquifers (Equation 6).

Discussion

Discretization Error

The "Discretization Error" (DE) relates the number of sinks necessary to adequately describe the drawdown distribution and is a measure of the "relative smoothness" of the drawdown profile namely

$$\text{DE} = [\text{XS}/(\text{ns} - 1)]/\text{RP}_{\text{Ave}} \qquad (7)$$

where XS is horizontal projection of the well screen, ft, (m); RP_{Ave}, average distance from the point where drawdown is calculated and the point sinks, ft, (m) and ns, number of point sinks.

For most cases, a low DE($<$ 0.5) reflects a smooth drawdown profile.

Comparison of UDE with Analytical Equations for Horizontal Wells

Figure 4a and 4b show comparisons of the UDE for horizontal wells with the analytical equation developed by Hantush and Papadopulos (1962) for a collector well with one lateral. The drawdown profiles are parallel to the horizontal well in the x-direction with distances from the well being 10 ft (3.05 m), and 100 ft (30.48 m), respectively. The drawdown profile constructed using the Hantush-Papadopulos equation is shown by the black triangles and the drawdown profiles constructed using the UDE (Equation 6) by the solid lines. Three drawdown profiles (calculated using the UDE) are shown on each figure for 4, 10, and 20 sinks. As can be seen, the UDE using 10-point sinks provides a good approximation to the drawdown distribution calculated using the exact analytical equation.

Comparison of UDE to Numerical Models for Slant Wells

Figure 5a and 5b compare drawdown profiles for a slant well calculated using the UDE (Equation 6) to drawdown profiles calculated using a three-dimensional

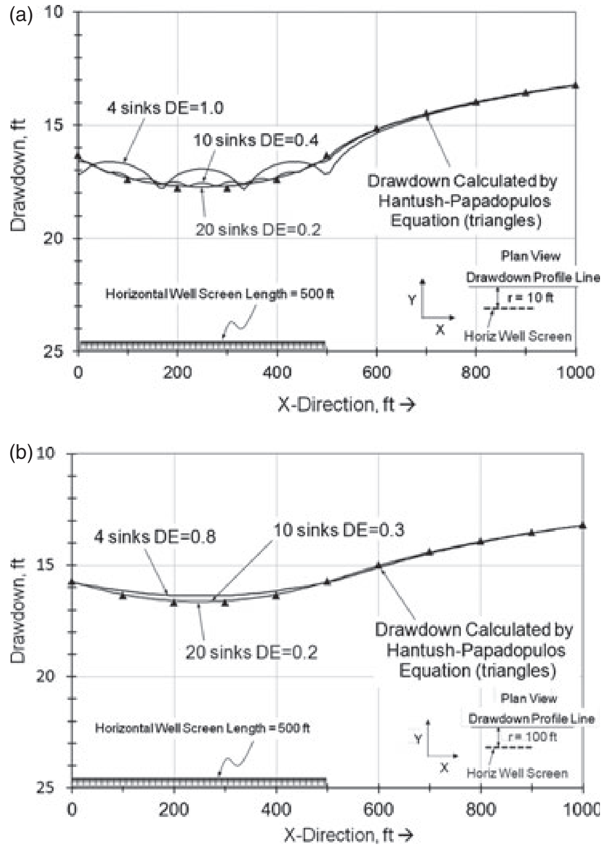

Figure 4. Comparison of UDE drawdown profiles, (ft) for a horizontal well with the Hantush-Papadopulos equation (a) profile 10 ft away and parallel to well, (b) 100 ft away. $\alpha = 0°$, $\beta = 90°$, L = LS=500 ft, K = 500 gpd/ft^2, $S_s = 1 \times 10^{-6}$/ft, b = 200 ft, Q = 1000 gpm, t = 365 d.

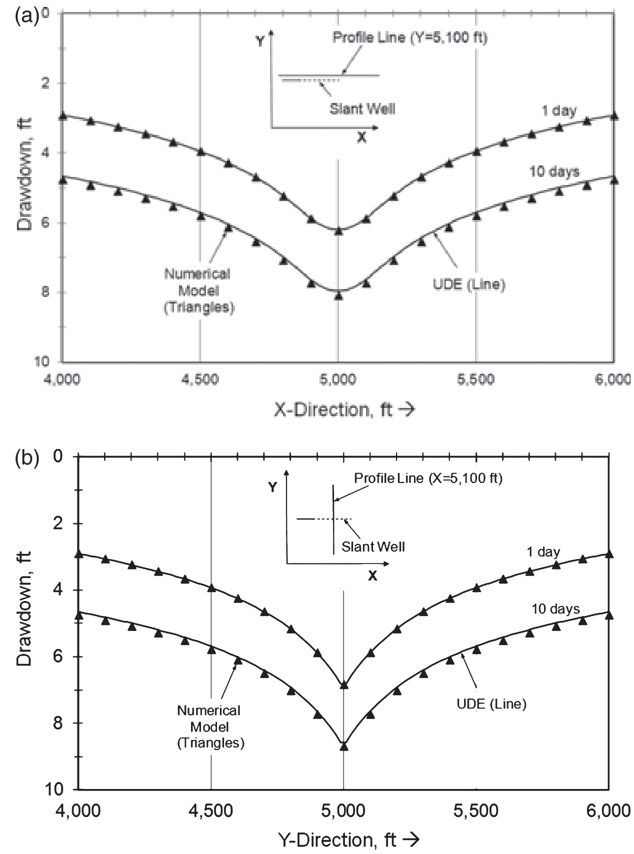

Figure 5. Comparison of UDE (Equation 6) drawdowns in the vicinity of a slant well with numerical model drawdowns. (a) Drawdown profile is parallel to the slant well axis and (b) profile is perpendicular to the axis ($\alpha = 45°$, $\beta = 90°$, L = 383 ft, LS = 283 ft, K = 750 gpd/ft^2, $S_s = 5 \times 10^{-6}$/ft, b = 200 ft, Q = 1000 gpm, XW = 4829 ft, YW = 5000 ft).

groundwater flow model. As can be seen, there is a good match between the UDE and numerical model drawdowns. The groundwater model consists of ten 20 ft (6.1 m) model layers with a total thickness of 200 ft (61.0 m). The 10-layer finite difference model grid covers a total area of 60 mi^2 (155 km^2) consisting of 500 rows and 500 columns. The smallest model cells are in the area surrounding the slant well and measure 5 ft (1.5 m) by 5 ft (1.5 m) and progressively increase toward the model boundaries reaching a maximum size of 6250 ft (1905 m). Well and aquifer parameters are listed in the caption for Figure 5. A total of 40 model cells were used to simulate the slant well pumping (four cells per layer). Stress period length was 0.1 d for the first 10 stress periods, and 9 d for the last stress period. The head change criterion and residual criterion for convergence were 0.0005 ft (1.5×10^{-4}m) and 0.000116 ft^3/s (3.28×10^{-6} m^3/s), respectively. The computer code used was MODFLOW 2000 (Harbaugh et al. 2000), a block-centered, three-dimensional, finite-difference groundwater flow model developed by the U.S. Geological Survey. Groundwater Vistas, developed by Environmental Simulations, Inc., was used for pre- and postprocessing of model data.

Sensitivity of vertical leakance (i.e., MODFLOW VCONT parameter) between the ten numerical model layers showed the same average drawdowns along the X-Y profiles for horizontal/vertical conductivity ratios (K_h/K_v) varying from 1:1, 20:1, 100:1, 500:1, and 1000:1.

Comparisons between the UDE solution and numerical model solution were also made for a slant well pumping near a constant head boundary (i.e., stream). The drawdown distribution calculated using the UDE (Equation 6) utilized recharging image wells (i.e., point sources) for each of the slant well sinks to simulate the constant head boundary created by the stream (Figure 6a). The drawdown distribution calculated using a numerical groundwater model is shown in Figure 6b. A ten layer groundwater model with a constant head boundary was used to create the model profile shown in Figure 6b. As can be seen, there is a close match between the UDE and Figure 5a numerical model results. When streambed conductance is reduced, a head loss occurs through the semi-pervious stream channel bed resulting in higher drawdowns in the immediate vicinity of the stream. In this case, the UDE image wells would need to have a partial strength proportional to the head loss incurred by the semipervious stream channel (Roscoe Moss Co. 1990).

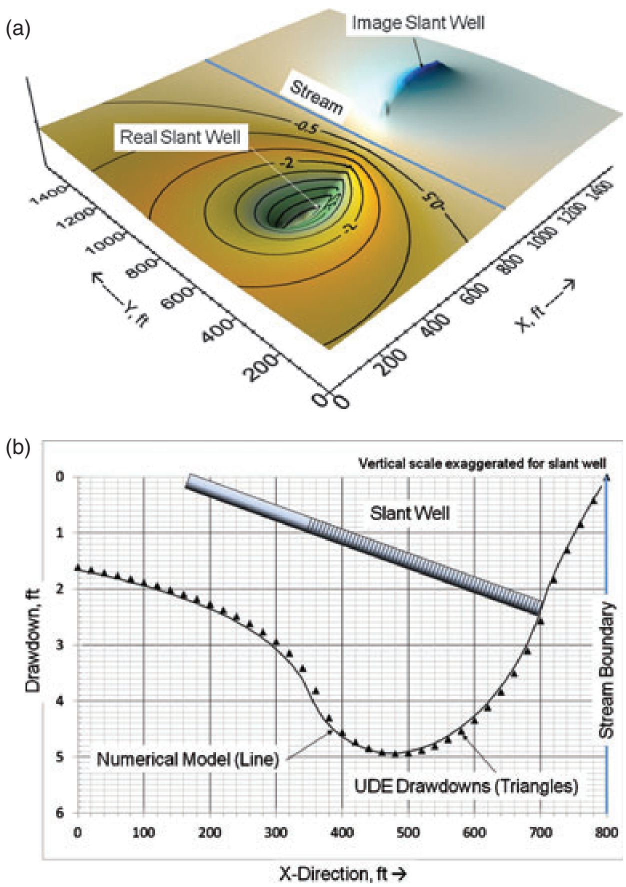

Figure 6. Slant well drawdowns near a stream. (a) Real and image well drawdowns using the UDE (Equation 6). (b) Cross section through $Y = 610$ ft comparing UDE drawdowns to numerical model drawdowns. ($\alpha = 30°$, $\beta = 90°$, $L = 500$ ft, LS $= 400$ ft, $K = 500$ gpd/ft^2, $S_s = 1 \times 10^{-6}$/ft, $b = 200$ ft, $Q = 1000$ gpm, $t = 10$ d, ns $= 10$).

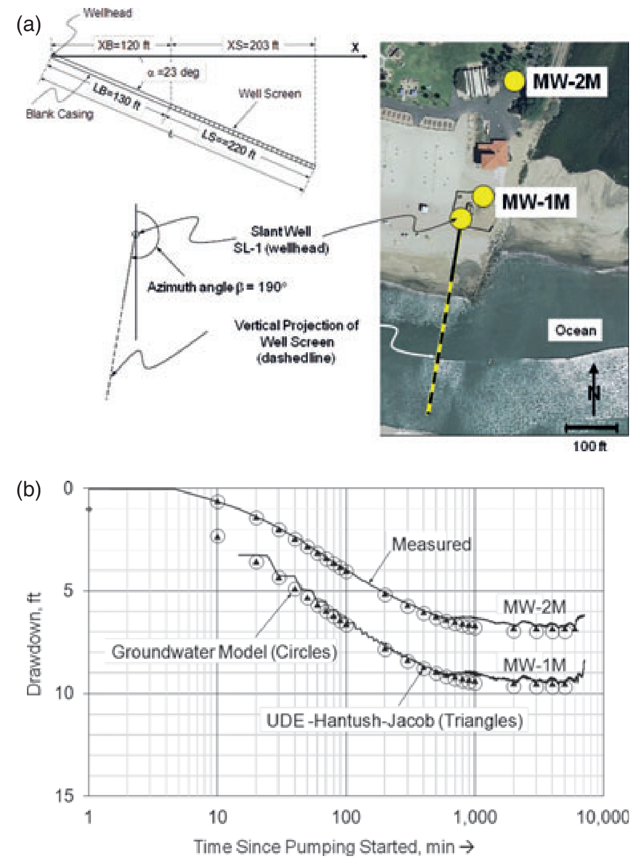

Figure 7. Dana Point slant well location and orientation (a). (b) Measured, modeled and UDE-calculated drawdowns for MW-1M and MW-2M (α=23°, β=190°, $L = 350$ ft, LS $= 220$ ft, $K = 426$ gpd/ft^2, $S_s = 5.85 \times 10^{-6}$/ft, $b = 200$ ft, $K'/b' = 0.003$/d, $Q = 1660$ gpm, ns $= 4$).

Slant Wells Pumping Beneath the Ocean Floor – Dana Point Test Slant Well

In 2006, a 350-ft long 12 in. diameter slant well was constructed on Doheny beach extending offshore into subsea aquifers. Subsurface deposits of sand and gravel associated with the San Juan Creek channel extending offshore beneath the ocean provide excellent filtration for a subsurface feed water supply (Williams 2008). Initial testing included a 5-d pumping test followed by an 18-month pumping period starting in June 2010. Data from the pumping tests are being used to evaluate subsea aquifer properties and feed water salinity parameters which will be used to design a full-scale 30 mgd feed water supply using nine 1000 ft slant wells (Williams 2011). Figure 7a shows the layout of the Dana Point test slant well (SL-1) and two nearby vertical observation wells (MW-1M) and (MW-2M). The observation wells are fully screened in the producing aquifer of the slant well. Due to the presence of a semi-pervious layer generally existing in the benthic zone on the ocean floor, a slant well producing from a subsea aquifer behaves exactly like a well in an infinite leaky aquifer with a constant head source (ocean). If it is also assumed that the majority of recharge to the slant well occurs from induced infiltration through the sea floor, the UDE incorporating the Hantush-Jacob leaky aquifer equation can be used to calculate the drawdown distribution. Figure 7b shows a good match between drawdown measured in observation wells MW-1 and MW-2 and drawdowns calculated using the UDE for leaky aquifers. Also shown for comparison is the drawdown calculated using a numerical groundwater flow model with the same well and aquifer parameters as used in the UDE. The numerical model simulated slant well drawdown with a number of point sinks in subsea aquifer model cells. Vertical leakance through the sea floor was simulated using a model leakance parameter (VCONT) of 0.003/d. Aquifer parameters were obtained from numerical model calibration results which were based on pumping tests, geophysical borehole logs and numerous onshore and offshore borings. The field test data preceded construction of the slant well and all data were used to calibrate a three-dimensional variable density groundwater flow and solute transport model. Further testing, onshore and offshore are planned to develop design parameters for the nine-well full-scale slant well feed water supply system.

Conclusions

A simplified method is presented to calculate the drawdown distribution in the vicinity of nonvertical wells. Using the principle of superposition combined with standard well hydraulics, a number of equations are presented to calculate drawdown distribution in the vicinity of production wells with inclination angles below horizontal ranging from 0° (horizontal wells) to 90° (vertical wells). The method is computationally simple and other than the normal assumptions for standard well equations, only requires that the calculated drawdown represent that which would be measured in a fully penetrating observation well. These angled well drawdown equations (UDE) utilize the principle of superposition to add incremental drawdowns from a discrete number of point sinks placed within the vertical projection of the well screen. The number of sinks required to provide an accurate drawdown primarily depends on the length of the vertical projection of the well screen (a function of the well's inclination angle), and the radial distance to the point where drawdown is calculated. The universality of the solution lies in the fact that it utilizes proven well hydraulic equations for steady and nonsteady flow regimes and for confined, semi-confined and unconfined aquifer systems. Computationally simple and straight forward, the drawdown calculated by the UDE provides a close match to analytical solutions, numerical models and field test data from the Dana Point slant well in Southern California.

Acknowledgments

The author is grateful to the editor and reviewers' helpful comments. Special appreciation also goes to Dr. Johnson Yeh of GEOSCIENCE for his invaluable technical assistance and modeling support.

Supporting Information

Additional Supporting Information may be found in the online version of this article:

The following supporting information is available for this article:

Appendix S1. UDE Development for Steady-State Flow Problems.

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